Scheme of Examination for M.Sc. Mathematics
Through Distance Education
with effect from the session 2013-14

M.Sc. (Previous) Mathematics

1. There will be five theory papers in each year.
2. Each theory paper will consist of four sections.
3. Paper setter will set ten questions.
4. The candidate will be required to attempt five questions in all, selecting at least one from each section. All questions will be of equal marks.
5. Duration of examination of each theory paper will be three Hours.
6. Max. Marks of each theory paper will be 100, 80 Marks for External Theory Examination and 20 Marks for Internal Assessment.
Scheme of Examination for M.Sc. Mathematics
Through Distance Education
with effect from the session 2013-14

M.Sc. (Previous) Mathematics

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<th>Paper</th>
<th>Code</th>
<th>Subject</th>
<th>Marks</th>
<th>Time</th>
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<td>MM-401</td>
<td>Advanced Abstract Algebra</td>
<td>80</td>
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<td>II</td>
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<td>Real Analysis</td>
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<td>MM-403</td>
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<td>Differential Equations</td>
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Syllabus for M.Sc. (Previous) Mathematics
Through Distance Education with effect from the session 2013-14

Paper-I: MM 401 Advanced Abstract Algebra

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

NOTE
The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Two Questions):

Section-II (Three Questions):

Section-III (Two Questions):

Section-IV (Three Questions):

Recommended Text:
References:

Syllabus for M.Sc. (Previous) Mathematics
Through Distance Education For the session 2013-14

PAPER-II : MM 402 Real Analysis

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

NOTE
The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):
Definition and existence of Riemann-Stieltjes integral, properties of the Integral, Integration and differentiation, the fundamental theorem of Calculus, integration of vector-valued functions, Rectifiable curves. (Scope as in Chapter 6 of ‘Principles of Mathematical Analyses’ by Walter Rudin (3rd edition)
Rearrangements of terms of a series, Riemann’s theorem,(Scope as in 3.52) to 3.55 of chapter 3 of Principle of Mathematical Analysis’ by Walter Rudin (3rd Edition)
Sequences and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel’s and Dirichlet’s tests for uniform convergence, uniform convergence and Continuity, uniform Convergence and Riemann-Stieltjes integration uniform convergence and Differentiation, Weierstrass Approximation Theorem. Power Series, Uniqueness theorem for power series, Abel’s and Tauber’s theorems. (Scope as in 7.1 to 7.27 of Chapter 7 of ‘Principles of Mathematical Analysis’ by Walter Rubin (3rd Edition) and 8.1 to 8.5 of chapter 8 of Principles of Mathematical Analysis’ by Walter Rudin (3rd Edition).

Section –II (Two Questions):

Section-III (Two Questions):

Section-IV (Three Questions)

**Recommended Texts:**

**References:**
1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975
5. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., Published New Delhi, 1986 (Reprint 2000).
11. P.R. Halmos, Measure Theory, Van Nostrand, Princeton 1950
Syllabus for M.Sc. (Previous) Mathematics
Through Distance Education For the session 2013-14
PAPER-III : MM 403 Topology And Functional Analysis

External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

NOTE
The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):
Definition and examples of topological spaces. Neighbourhoods, ulterior point and interior of a set. Closed set as a complement of an open set. Adherent point and limit point of a set, closure of a set as a adherent points, derived set of a set, properties of closure operator, boundary of a set, Dense subsets.
Base and sub-base for a topology. Neighbourhood system of a point and its properties. Base for Neighbourhood system.
First countable. Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and second countable space, Lindelof.. theorems.
Comparison of Topologies on a set, About intersection, union, infimum and supremum of a collection of topologies on a set.
Definition, examples and characterizations of continuous functions, composition of continuous functions. Open and closed functions, Homoeomorphism.
Tychonoff product topology in terms of standard subbase, projection maps. Characterisation of product topology as smallest topology with projections continuous, continuity of a function from a space into a product of spaces, countability and product spaces.
To, T1, T2 Regular & T3, separation axioms, their characterization and basic properties i.e. hereditary property of To, T1, T2 Regular & T3 spaces, and productive property of T1 & T2 spaces.
Quotient topology w.r.t. a map. Continuity of function with domain a space having quotient topology. About Hausdorffness of quotient space.
Completely regular and Tychonoff (31/2), spaces, their hereditary and productive properties, Embedding lemma. Embedding theorem.

SECTION II (Two Questions)
Normal and T4 spaces: Definition and simple examples. Normality of a regular Lindel of space, Urysohn’s Lemma, complete regularity of a regular normal space, T4 implies Tychonoff, Tietze’s extension theorem.
Filter on a set : Definition and examples. Collection of all filters on a set as a P.O. set, finer filter, methods of generating filters/finer filter. Ultra filter (u.f.) and its characterizations. Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.
Compactness : Definition and examples of compact spaces and subsets, compactness in terms of finite intersection property, continuity and compact sets, compactness and separation properties,
closedness of compact subset and a continuous map from a compact space into a Hausdorff and its consequence. Regularity and normality of a compact Hausdorff space.

Compactness and filter convergence, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse. (both the sections i.e. I & II are based on Chapters 1 to 5 of the Book General Topology by John L. Kelley).

SECTION III (Three question)
Normed Linear spaces, Banach spaces, completion of a normed space, finite dimensional normed spaces and subspaces, equivalent norms, F.Riesz’s Lemma, bounded and continuous linear operators, normed spaces of operators, dual spaces.

Hahn Banach theorem, application to bounded linear functionals on C[a,b], adjoint operator, reflexive spaces, uniform boundedness theorem.

Strong and weak convergence, open mapping theorem, bounded inverse theorem, closed linear operators closed graph theorem.

(Scope as in relevant parts of Chapters 2 & 4 of “Introductory Functional Analysis with applications” by E.Kreyszig)

SECTION IV (Two question)
Inner product spaces, Hilbert spaces and their examples, Pythagorean theorem, Appolloniu’s identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct sums, projection theorem.

Orthonormal sets and sequences, Bessel’s inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval’s identity, separable Hilbert spaces. Representation of functionals on Hilbert spaces, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space.

Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self, adjoint, unitary, normal, positive and projection operators. (Scope as in relevant parts of Chapters 3 & 9 of “” by E.Kreyszig.)

Books


2. E.Kreyszig Introductory Functional Analysis with applications(John Willey 1973)
NOTE
The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):

Section-II (Two Questions)
Residues. Cauchy’s residue theorem-Evaluation of integrals. Branches of many valued functions with special reference to agr z, log z and $z^a$
Bilinear transformations, their properties and classifications. Definitions and examples of conformal mappings.

Section-III (Three Questions):
Spaces of analytic functions. Hurwitz’s theorem, Montel’s theorem, Riemann mapping theorem.

Section-IV (Two Questions)
Univalent functions. Bieberbach’s conjecture (statement only) and the theorem.

RECOMMENDED TEXT:
Reference:
4. S. Lang, Complex Analysis, Addison Wesley, 1977
5. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994
7. E.Hille, Analytic Function Theory (2 Vols.) Gonn & Co., 1959
12. S.Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952
16. Syllabus for M.Sc. (Previous) Mathematics
Through Distance Education For the session 2013-14

Paper-V: MM 405 Differential Equations
External Theory Marks: 80
Internal Assessment Marks: 20
Time: 3 Hours

NOTE
The examiner is requested to set ten questions by dividing the paper into the sections as indicated in the syllabus. The candidates are required to attempt five questions, selecting at least one from each section.

Section-I (Three Questions):
Preliminaries: Initial value problem and equivalent integral equation, \( \varepsilon \)-approximate solution, equicontinuous set of functions.
(Relevant portions from the book of ‘Theory of Ordinary Differential Equations’ by Coddington and Levinson)
(Relevant portions from the book ‘Ordinary Differential Equations’ by P. Hartman)

Section-II (Two Questions):
   Linear differential systems: Definitions and notations. Linear homogeneous systems; Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Non-homogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.
(Relevant portions from the book of ‘Theory of Ordinary Differential Equations’ by Coddington and Levinson)
   Higher order equations: Linear differential equation (LDE) of order \( n \); Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel’s Identity, Fundamental set, More Wronskian theory.
   Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.

Section-III (Two Questions):
   Autonomous systems: the phase plane, paths and critical points, Types of critical points; Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications.
Section-IV (Three Questions):


Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and their corollaries.


Recommended Text:

2. S.L. Ross, *Differential Equations*, John Wiley & Sons

References: